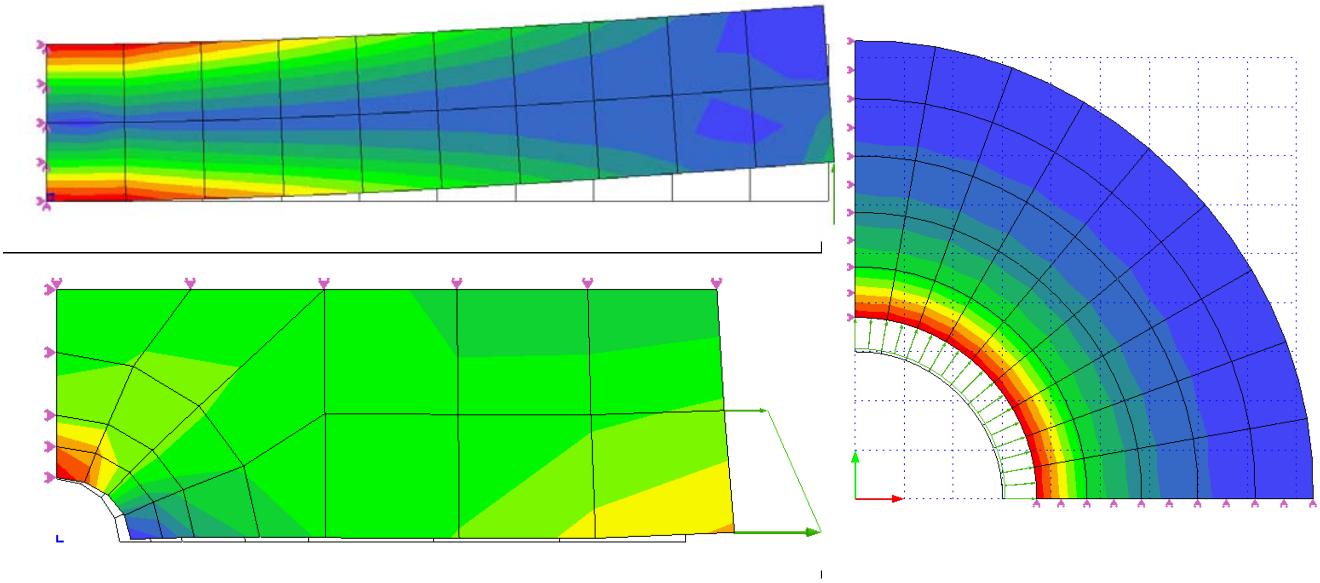
# 1. REPORT

## NAFEM'S "WORKBOOK OF EXAMPLES"

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CONTENTS

# 1 EXERCISE 4: CONTINUMM BEAM – TIP LOAD

## 1.1 Input data

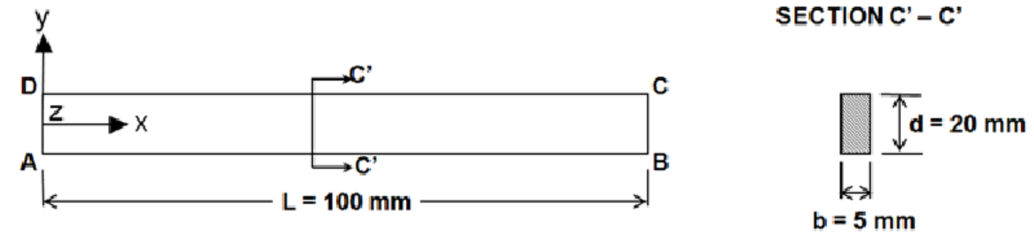


*figure 1-1 (Input data)*

| F[N] | E[MPa] | $\nu$ |
|---|---|---|
| 1000 | 200000 | 0,3 |

## 1.2 Result

### 1.2.1 Mesh overview

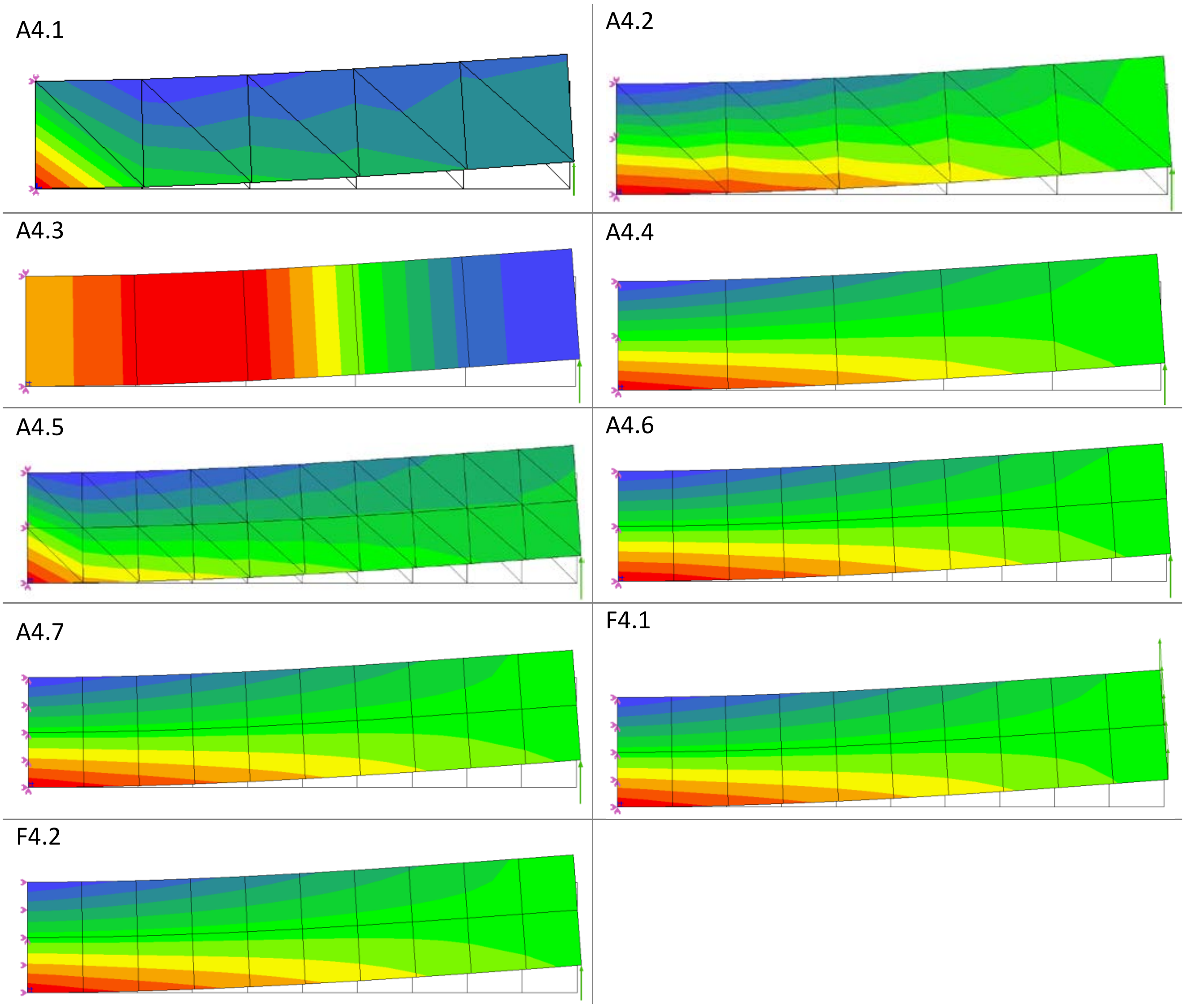


*figure 1-2 (mesh overview – sigma (x) )*

The figure 1-2 (mesh overview – sigma (x)) shows the different meshes and the Result (sigma xx) of Trinitas Calculation.

### 1.2.2 Displacement

- Analytical solution

The Displacement in Y direction can be calculated analytically by using the following equation of Timoshenko:

*moment of inertia:* $I_{(z)} = \frac{bd^3}{12} = 3333.333\text{mm}^4$

*Beam bending contribution:* $y_{max} = \frac{F \cdot l^3}{48 \cdot E \cdot I} = 0.5mm$

*shear modulus:* $G = \frac{E}{2(1+\nu)} = 76923MPa$

*Cross − sectional area:* $A = b \cdot d = 100mm^2$

*Shear deflection:* $\delta_{shear} = 1{,}2 \cdot \frac{F \cdot x}{A \cdot G} = 0{,}0156mm$

***Transverse Deflection***$_{max}$ $= \boldsymbol{y_{max} + \delta_{shear} = 0{,}5156mm}$

***Transverse Deflection***$_{(x)}$: $\boldsymbol{y_{(x)} + \delta_{(x)} = F \cdot \frac{l^3}{6 \cdot E \cdot I} \cdot \left(\frac{3 \cdot x^2}{l^2} - \frac{x^3}{l^3}\right)} + 1{,}2 \cdot \frac{F \cdot x}{A \cdot G}$

- Diagram

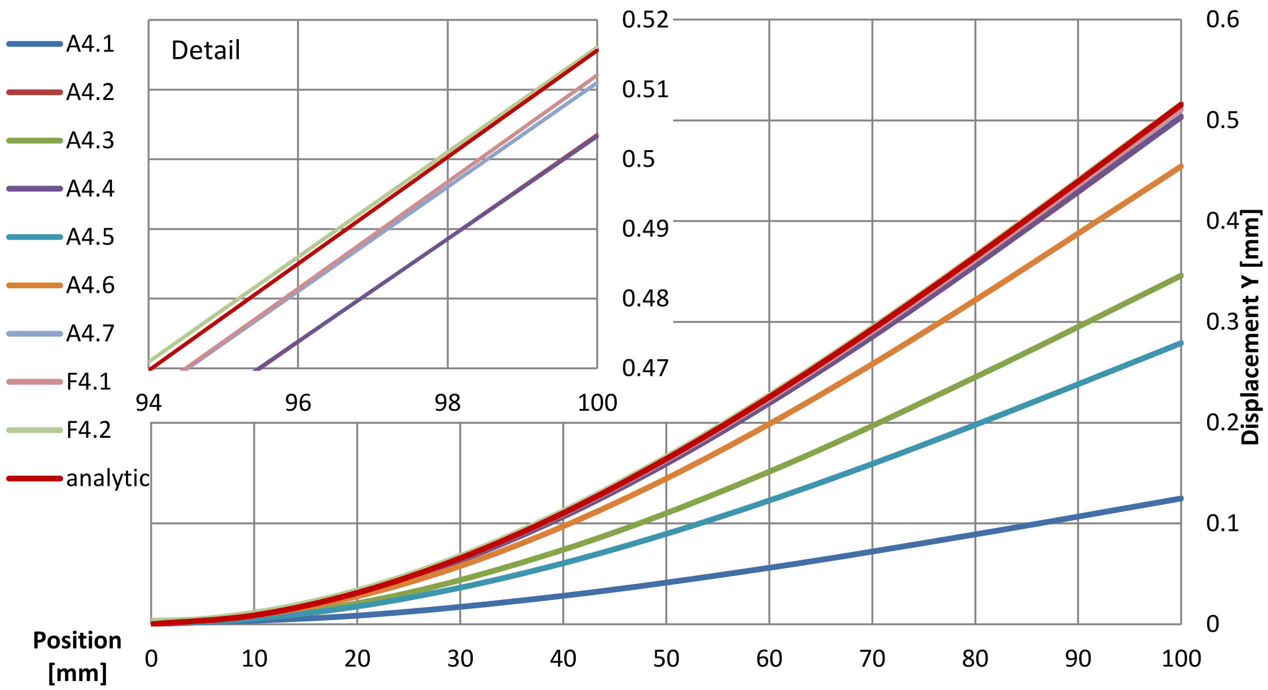


*figure 1-3 (displacement in Y-direction)*

### 1.2.3 Bending stress

- Analytical solution

The bending stress can be calculated analytically by using the following equation:

Moment of inertia: $I_y = \frac{b \cdot d^3}{12} = 3333.333\text{mm}^4$

bending moment: $M = F \cdot l = 100000\text{Nmm}$

The linear bending stress over the beam: $\sigma_{bmax} = \frac{M}{I_y} \cdot \frac{d}{2} = 300 \frac{N}{mm^2}$

The maximal bending stress at the beam: $\sigma_{(x)} = \frac{-F * (L - x)}{I_y} \cdot \frac{d}{2}$

- Diagram

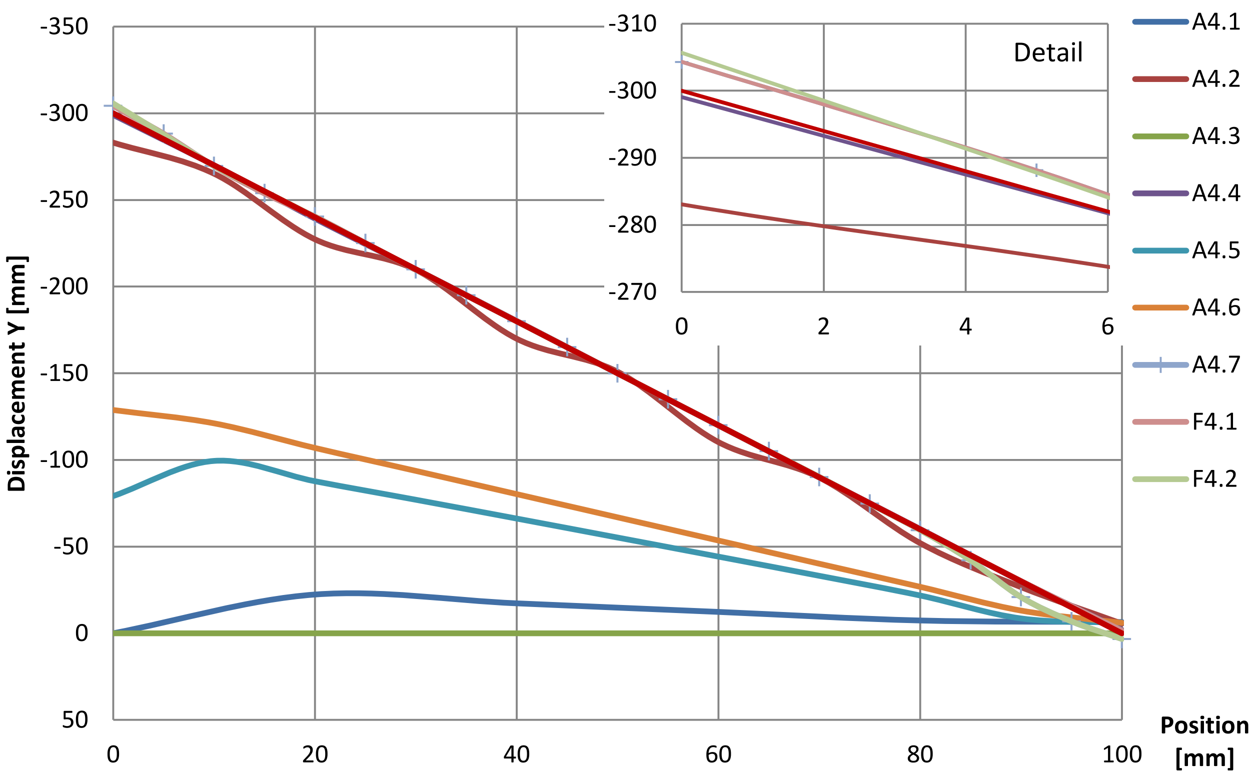


*figure 1-4 (Bending stress)*

### 1.2.4 Shear stress

- ANALYTICAL SOLUTION

The shear stress can be calculated by using the general equation:

$$\tau_{(xy)} = \frac{3}{2}\frac{Q}{bd}\left(1 - \left(\frac{2y}{d}\right)^2\right)$$

→Q is the force in y-direction

The maximal shear stress is in the middle of the cross section, as you can see in **Error! Reference source not found.**.

$$\tau_{max} = \tau_{(y=0)} = \frac{3}{2}\frac{Q}{bd} = 15$$

The shear stress at the upper and under edge of the beam:

$$\tau_{\left(y=\frac{d}{2}\right)} = 0$$

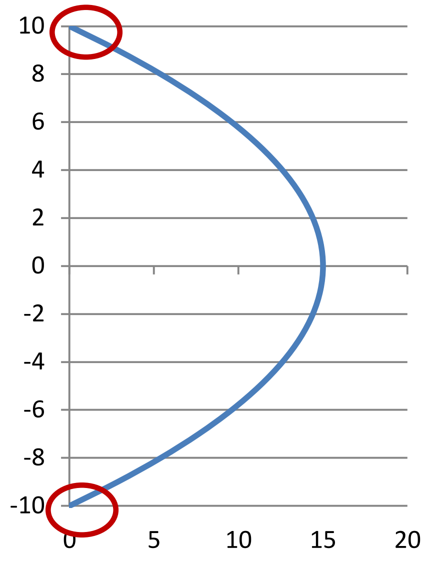


figure 1-5 (shear stress in Y-direction)

- Diagram

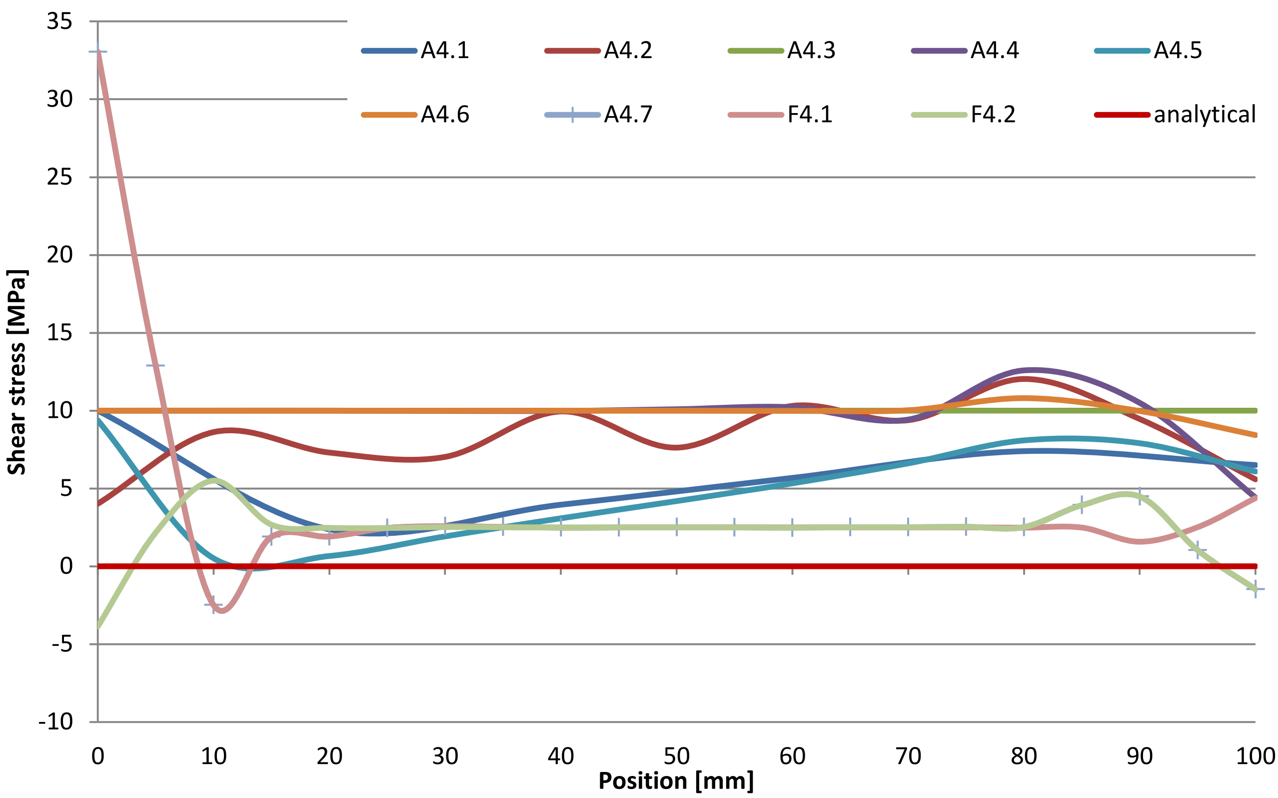


*figure 1-6(shear stress)*

## 1.3 Discussions

### 1.3.1 A4.1 – 3 noded triangles element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,124mm | **75,8%** |
| Stress | 300MPa | fail | |

The combination of the low mesh density and the element type lead to a different of 75,8% between the Analytical-, and the Trinitas-displacement. These CST-Elements are too stiff, because this element uses only the linear shape function. This effect is called volume locking and is the reason of the huge different between the displacements of this solution. If is the displacement wrong, it's impossible to get a correct solution.

### 1.3.2 A4.2 – 6 noded triangles element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,5036mm | **2,33%** |
| Stress | 300MPa | 283 | **5,65%** |

Their significance improvement in the result obtains for both displacement and stress. Only 2.33% difference in the displacement and 5,65% difference in the stress. The solution has also significantly improved, because the element formulation with quadratic Shape function leads to better solutions than in the previously (A4.1) with 3 nodes.

### 1.3.3 A4.3 – 4 noded quadrilateral element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,3459mm | **32,9%** |
| Stress | 300MPa | fail | |

This displacement in this Solution is better than the 3 node Triangular element, because the order of the linear Shape functions/dof[1] is higher, which lead to better solution in displacement. The stress result is still fail completely.

### 1.3.4 A4.4 – 8 noded quadrilateral element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,5032mm | **2,39%** |
| Stress | 300MPa | 299,06MPa | **0,32%** |

In this case the displacement and the stress are quite the same as in the analytical solution. The stress-performance of the 8-node element with quadratic shape functions is even better than an element with a linear shape function. Therefore these elements have a bilinear stress fields. This means that is possible to receive linear stress in both directions.

[1] Dof= degrees of freedom

### 1.3.5 A4.5 – 3 noded triangles element with double number of elements

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,279mm | **45,89%** |
| Stress | 300MPa | 79,17MPa | **73,6%** |

The result is better than in A4.1, because the mesh density has been increased to 4 times more elements. But the results are still bad. The displacement has a difference of 45.89% to the analytical solution. Based on the displacement, the stress is still bad. To get a good solution with this element type, many elements would be need.

### 1.3.6 A4.6 – 4 noded triangles element with double number of elements

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,454mm | **11,88%** |
| Stress | 300MPa | 128,81MPa | **57,06%** |

The result is better than in A4.2, because the mesh density has been increased. This case has a similar effect as this one in the A4.5. But the ratio to displacement is better than the stress difference. It is still a bad solution with a difference in stress of 57.06%. Here is the same problem with the linear shape function as discussed above.

### 1.3.7 A4.7 – 8 noded triangles element with double number of elements

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,511mm | **0,9%** |
| Stress | 300MPa | 304,3MPa | **1,44%** |

The result is better than the solution in A4.4. As you can see in the table above, the difference between Trinitas and the Analytical solution are quit close.

### 1.3.8 F4.1 – 8 noded triangles element with double number of elements

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,512mm | **0,69%** |
| Stress | 300MPa | 304,3MPa | **1,44%** |

The stress difference is the same as in A4.7, but the difference in displacement is better as in the solution above. The load line leads to a better stress distribution, because the load is not applied to a single node. Note that a point load can lead to a singular place.

### 1.3.9 F4.2 – 8 noded triangles element with double number of elements

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Displacement | 0,5156mm | 0,516mm | **0,09%** |
| Stress | 300MPa | 305,67MPa | **1,89%** |

The boundary conditions in this simulation are changed. The new boundary condition is very good for bending problems, because the fixing points on the outer surface are only fixed in the x-direction and the transverse strains are allowed. The different between the displacements of 0.9% are very good.

### 1.3.10 Summary

- The stresses at the linear shape function can't be accurate as long the displacement is incorrect.
- Triangular 6 nodded elements give a better result than triangular 3 nodded elements
- Quadrilateral elements give a better result than triangular elements
- 8-noded Quadrilateral elements give a better result than 4 nodded Quadrilateral elements
- Not only the number of elements is important. Furthermore it can be possible, that the result of a better element type gets closer to the analytic solution than the result with the huge mesh density.

## 2 EXERCISE 5: INFINITE THICK PIPE

### 2.1 Input Data

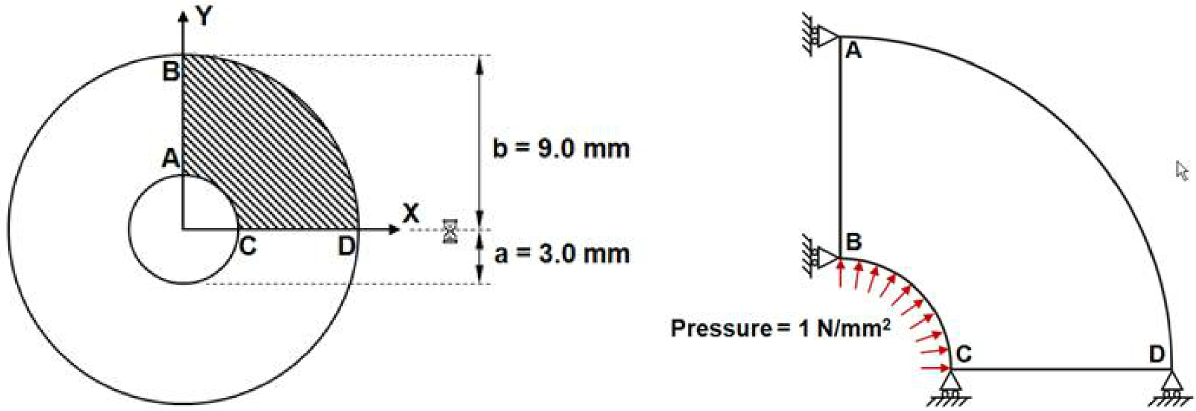


*figure 2-1(input data)*

### 2.2 Result

#### 2.2.1 Mesh overview

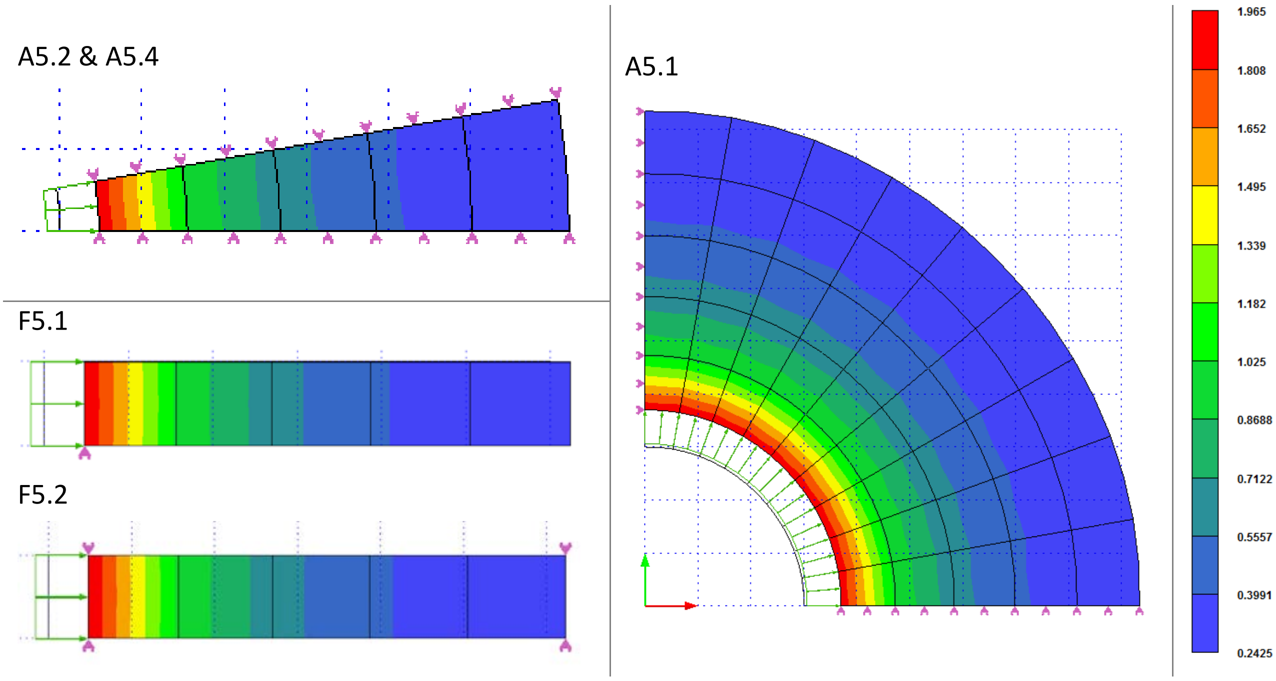


*figure 2-2 (mesh overview)*

### 2.2.2 Radial Stress

- Analytical solution

$$\text{Radial Stress: } \sigma_r = -p + \frac{\left(\frac{r_a}{r}\right)^2 - 1}{\left(\frac{r_a}{r_i}\right)^2 - 1}$$

- Diagram

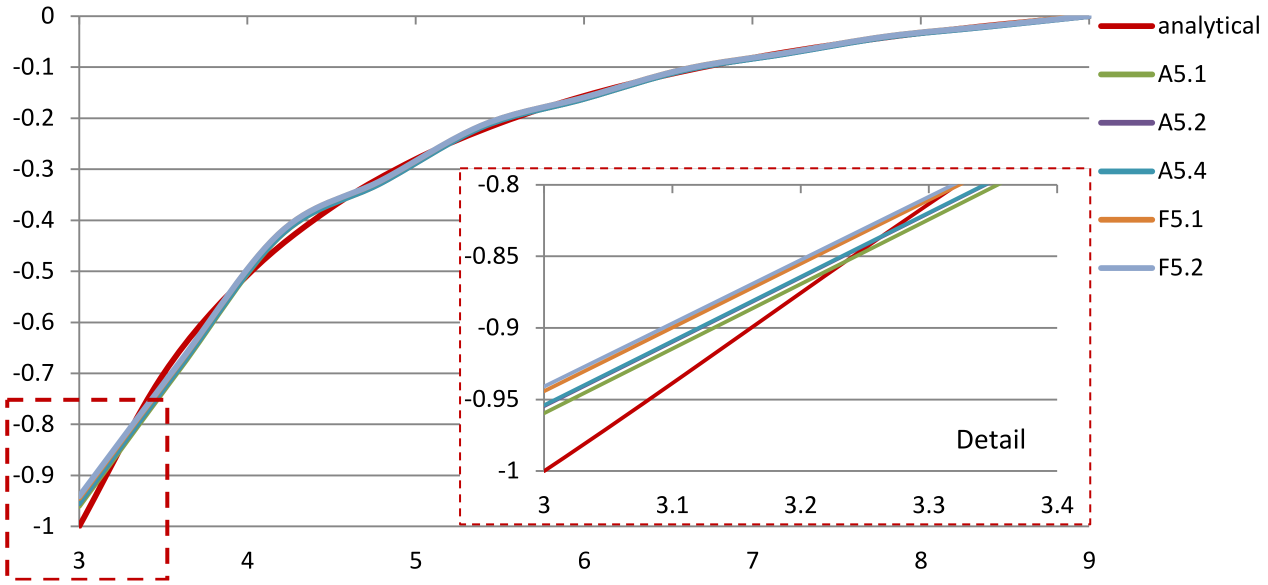


*figure 2-3 (radial stress)*

### 2.2.3 TANGENTIAL (hoop) STRESS

- Analytical solution

$$Tanential\ Stress: \sigma_t = p + \frac{\left(\frac{r_a}{r}\right)^2 + 1}{\left(\frac{r_a}{r_i}\right)^2 - 1}$$

- Diagram

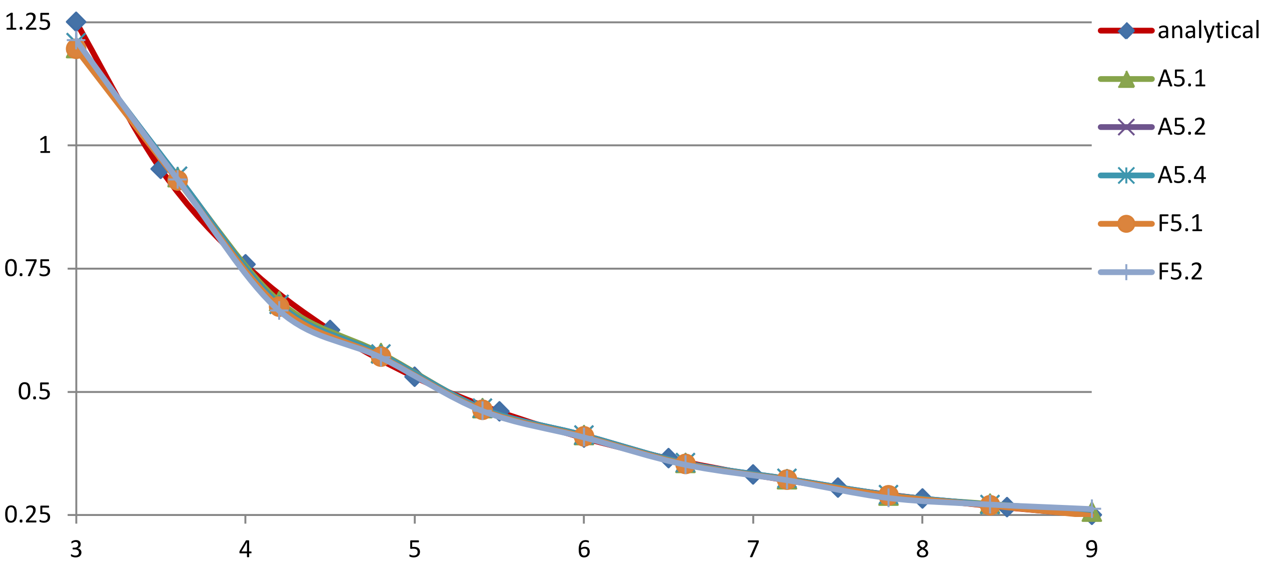


*figure 2-4 (tangential stress)*

### 2.2.4 RADIAL DISPLACEMENT

- Analytical solution

$$Q = \frac{R_i}{R_o} \qquad U_{(r)} = \frac{p}{E} \cdot \frac{\left[(1-v) \cdot Q^2 \cdot r + (1+v) \cdot \frac{r_i^2}{r}\right]}{1-Q^2}$$

- Diagram

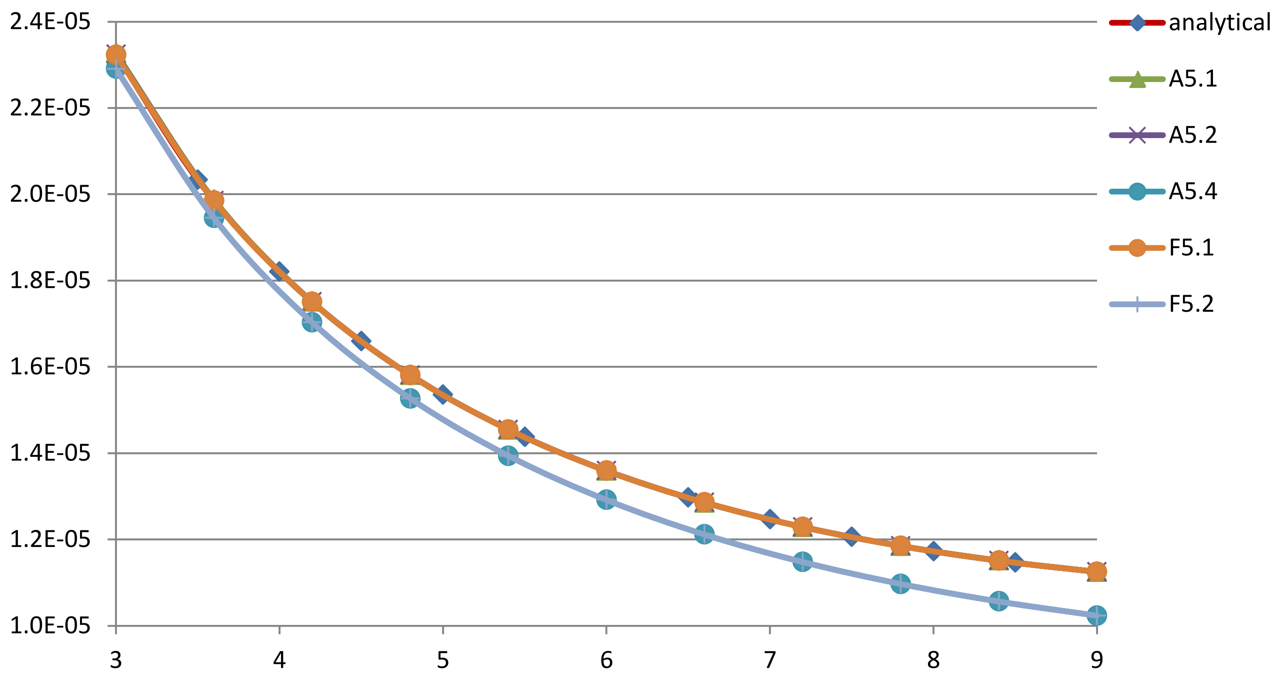


*figure 2-5 (radial displacement)*

## 2.3 Discussions

### 2.3.1 A5.1 - quarter section of the pipe

The stress results are in good agreement with the theoretical values in displacements and stresses. Therefore the mesh density and the shape functions used in this case are satisfactory. However, results obtained for a refined mesh will converge to the theoretical curve.

### 2.3.2 A5.2 – 10° section of the pipe

In this case, the pipe reduced to 10° section and the results are the same like in the previous one, because the mechanical is not changed.

### 2.3.3 A5.4 – 10° section of the pipe with plain strain

The analysis used the plain strain definition. This produces the same stress in radial and tangential direction. The radial displacement is lower, because there is no Poisson effect.

### 2.3.4 F5.1 - axisymmetric elements

In this case is the axisymmetric elements used. The Results are the same as in the previously ones, because the mechanical problems are the same. This way of modeling is very fast, if there are no holes in the disc/pipe available.

### 2.3.5 F5.2 - axisymmetric elements with plain strain

This case simulates the plane strain, for that reason the displacement is the same as A5.4 (plain strain).

### 2.3.6 Summary

- It is important to find the right modeling type, because it save CPU time
- The mesh used in this exercise can be refined to obtain results that would converge to the analytical solution
- When refining the mesh it might be preferable to grade the mesh by biasing the number of elements towards the inner wall where the stress changes more rapidly

# 3 EXERCISE 9: FLAT BAR WITH EDGE NOTCHES

## 3.1 Input data

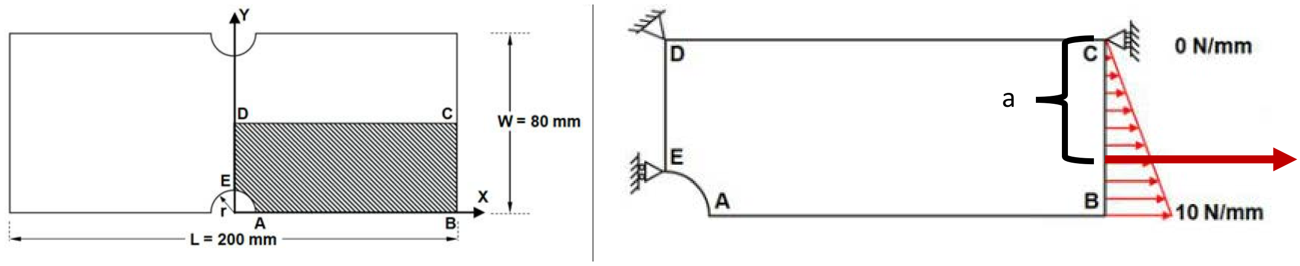


*figure 3-1 (input data)*

## 3.2 Results

### 3.2.1 Handbook Solution

$$F = \frac{10\frac{N}{mm} \cdot 40mm}{2} = 200N$$

$$a = \frac{2}{3} \cdot 40mm = 26{,}66mm$$

$$M = 200\text{N} \cdot 26{,}66\text{mm} \cdot 2 = 10666{,}67\text{Nmm}$$

$$\sigma = \frac{6M}{t(W-2r)^2}$$

$$= \frac{6 \cdot 10666{,}67\text{Nmm}}{1mm \cdot (80mm - 2 * 10mm)^2} = 17{,}78\frac{N}{mm^2}$$

See figure 3-2:

$$\left.\begin{aligned} \frac{B}{b} &= \frac{80}{80-2 \cdot r} = \frac{4}{3} \approx \infty \\ \frac{r}{b} &= \frac{10}{80-2 \cdot r} = \frac{1}{6} \end{aligned}\right\} \rightarrow K_t = 1{,}84$$

$$\sigma_{max} = \sigma \cdot K_t = 17{,}78\frac{N}{mm^2} \cdot 1{,}84 = \mathbf{32{,}7}\frac{\boldsymbol{N}}{\boldsymbol{mm^2}}$$

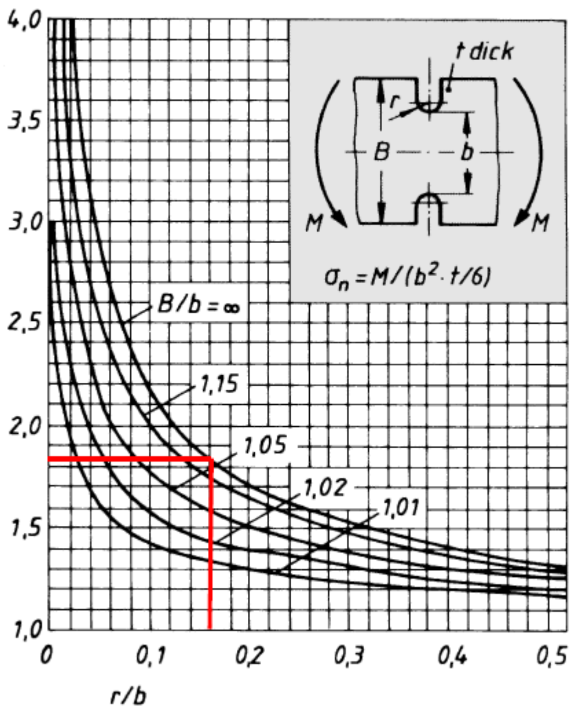


*figure 3-2 (Wittel, et al., 2009)*

### 3.2.2 Mesh

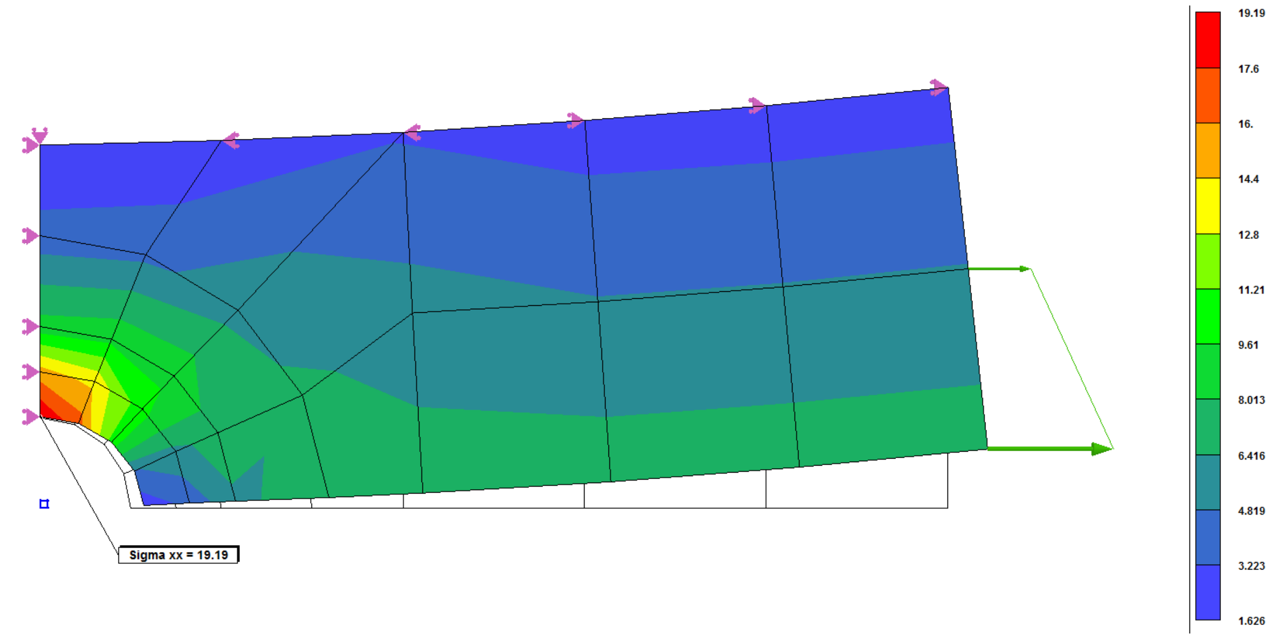


*figure 3-3 (4-noded element)*

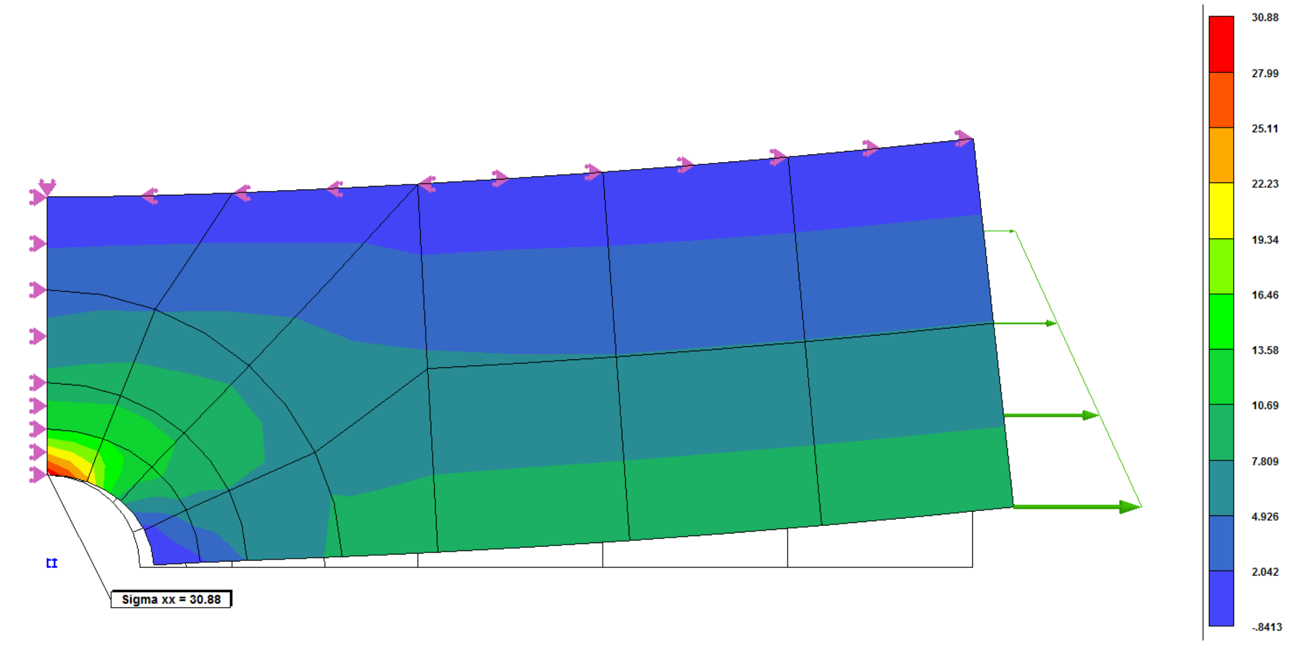


*figure 3-4 (8-noded element)*

The figures above show the different meshes and Results (von Mises) of Trinitas Calculation.

### 3.3 Discussions

#### 3.3.1 A9.1 - 4-noded quadrilateral element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Stress – 4 noded | 32,7MPa | 19,19MPa | **41,31%** |

The stress difference between Analytical and Trinitas is 43.31%. This Result obtained is poor and markedly less than the theoretical value due to the use of 4-noded elements with linear displacement shape functions.

#### 3.3.2 A9.2 - 8-noded quadrilateral element

| | Analytical | TRINITAS | % Difference |
|---|---|---|---|
| Stress – 8 noded | 32,7MPa | 30,88MPa | **5,57%** |

Here is the stress difference between Analytical and Trinitas only 5.57%.
This is an improved result compared to A9.1. Further mesh refinement would produce a result which is closer to the theoretical solution.

#### 3.3.3 Summary

- 8-noded quadrialeral elements give a better result than 4-noded quadrilateral elements
- In case of 4-noded elements a much higher mesh density is required to capture the stress field
  - ➔ In both cases results obtained will be closer to the theoretical value when the mesh is refined

# 4 APPENDIX

## 4.1 List of Figures



## 4.2 Bibliography

**Wittel, Herbert , et al. 2009.** *Roloff/Matek Maschinenelemente.* 2009. Vol. 18.