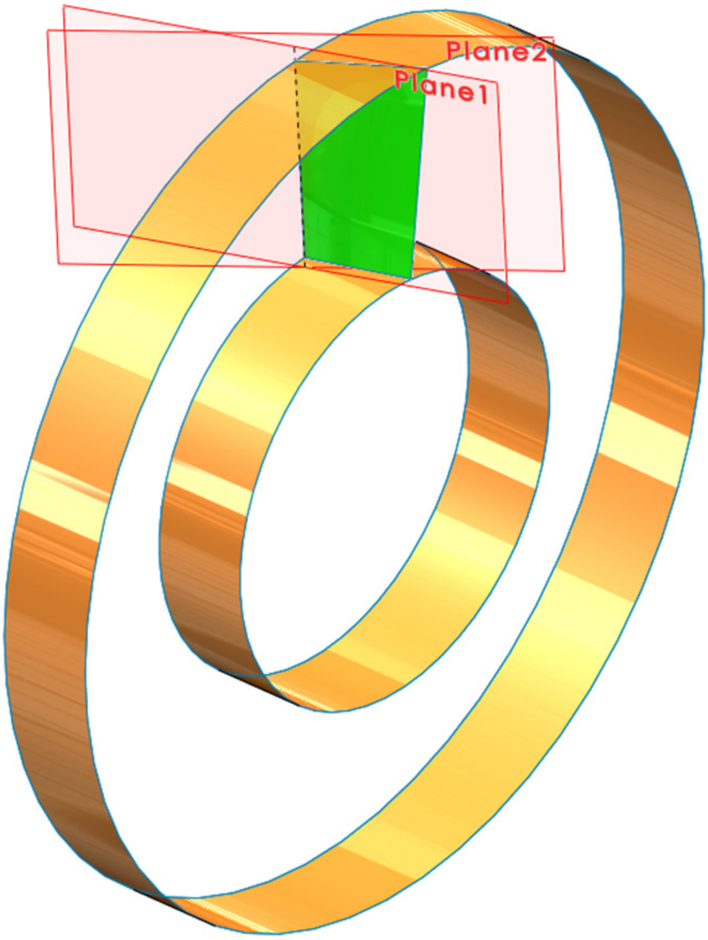
4. REPORT

DYNAMIC EIGEN-VALUE ANALYSIS



AUTHOR:

Stegmaier Alexander
[redacted]

Mechanical Engineering
TMHL62, the Finite Element Method, Advanced Course

EXAMINER:
Bo Torstenfelt
INSTITUTE OF TECHNOLOGY
SE-581 83 Linköping

CONTENTS

1 THEORETICAL EXPLANATION OF THE EIGEN-VALUE

1.1 Basics of Free Vibration

For a free vibration analysis, the natural circular frequencies ω_i and mode ϕ_i are calculated from:

$$\left([K] - \omega_i^2 [M]\right)\phi_i = 0$$

Where

$[K]$: stiffness matrix
ω^2 : eigenvalues
$[M]$: mass matrix
$\{\phi\}$: eigenvector

Assumptions:

- stiffness matrix [K] and mass matrix [M] are constant:
- Linear elastic material behavior is assumed
- Small deflection theory is used, and no nonlinearities included
- The structure can be constrained or unconstrained
- The Lanczos analysis are used

A modal analysis determines the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. You can also perform a modal analysis on a pre-stressed structure, such as a spinning turbine blade.

For a rotating structure, the gyroscopic effects resulting from rotational velocities are introduced into the modal system. These effects in turn change the system's damping. Such effects are commonly encountered in rotor dynamic analysis. The changes in Eigen characteristics at different rotational velocity can be studied with the aid of Campbell Diagram Chart Results.

2 COMPARE THE ANALYTICAL- WITH THE FEM-SOLUTION

2.1 Analytical-Solution by KTHs Handbook

Material properties: Young's Modulus, Poisson's Ratio, and Density are required.

Given Function:

$$\omega = \frac{\lambda}{a^2}\sqrt{\frac{D}{\rho \cdot h}} = \frac{Nmm}{\frac{kg}{mm^3} * mm}$$

where

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)} = Nmm$$

Assumed:

$$\frac{b}{a} = 1; \ \nu = 0.3$$

$$\lambda_1 = 3{,}494, \quad \lambda_2 = 8{,}547, \quad \lambda_3 = 21{,}44$$

parameters used:

$$a = 100mm, \quad b = 100mm, \quad h = 10mm$$

$$E = 103000MPa, \quad \rho = 4540\frac{kg}{m^3}$$

$$\rightarrow \omega_1 = 801, \quad \omega_2 = 1960, \quad \omega_3 = 4918$$

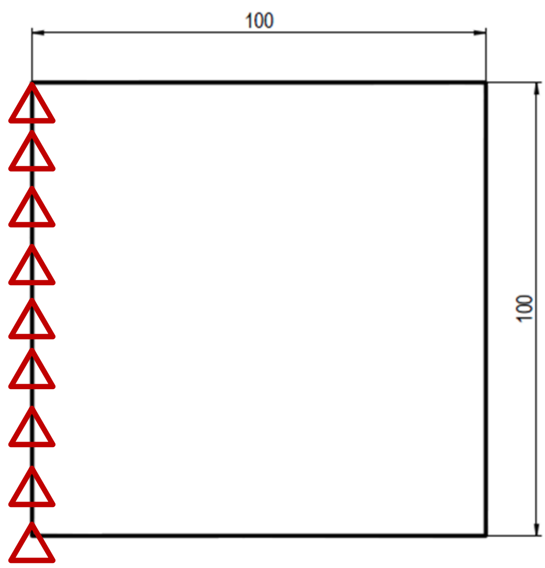


figure 2-1

2.2 FEM Solution with Ansys

2.2.1 Geometry, Mesh and BCs

The geometry is a surface with a thickness of 10mm. figure 2-2 shows the used mesh.

The Boundary Conditions are:

- One edge of the plate is fixed

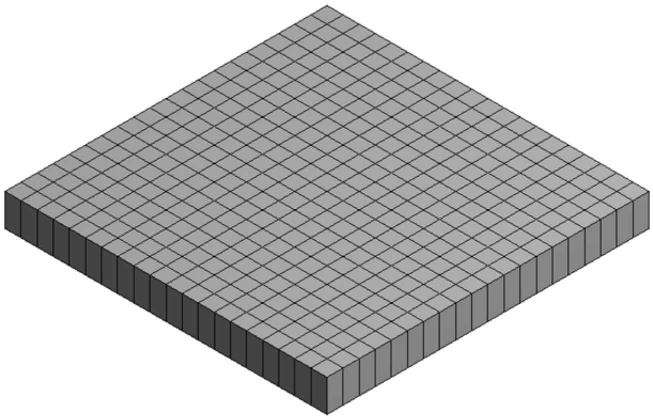
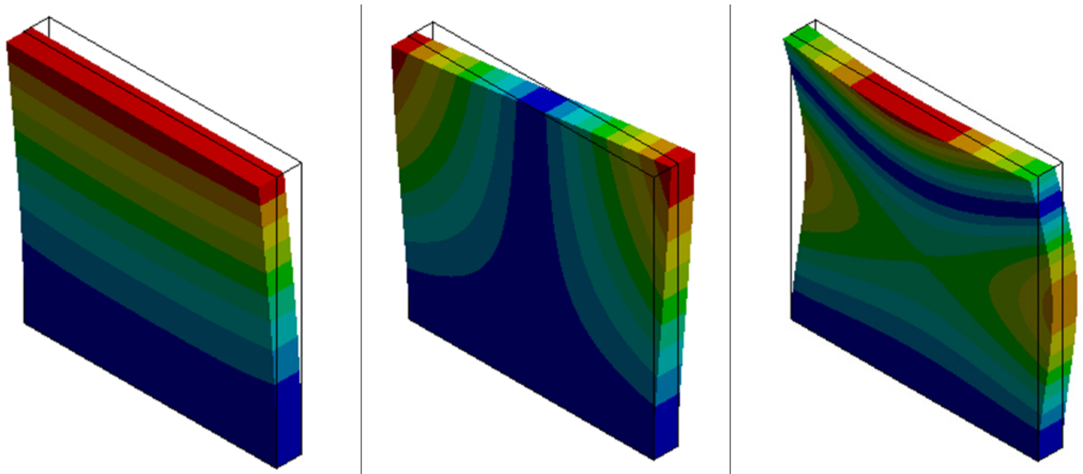


figure 2-2

2.2.2 Result

figure 2-3 shows the Results of the three first Eigen-values.



787Hz	1849Hz	4610Hz

figure 2-3

2.3 Comparison of analytical and FEM

In the following diagram (figure 2-3) and the Table 2-1 you can see the analytical- and the Ansys solution. Both solutions are nearly the same, thus the simulation can still be used.

Mode	Analytic	Ansys	Different [%]
1	801	787.04	1,7%
2	1960	1849.6	5.6%
3	4918	4610.2	6.6%

Table 2-1

3 DEF CAMPBELL

The first step in Ansys is a static structure to figure out the prestresses. Thus results are the input date for the Buckling analyses.

After you have run several modal analyses, you can perform a Campbell diagram analysis. The analysis allows you to:

- Visualize the evolution of the frequencies with the rotational velocity
- Check the stability and whirl of each mode
- Determine the critical speeds.

A Campbell diagram chart result is only valid in modal analyses. The Campbell diagram chart result is mainly used in rotor dynamics for rotating structural component design. When a structural component is rotating, an inertial force is introduced into the system. The dynamic characteristics of the structural component change as a result of the inertia effect, namely, gyroscopic effect. To study changes in dynamic characteristics of a rotating structure, more than one solve point in Rotational Velocity is required.

Four different ratios of 1EO-4EO show the critical places:

$$\_EO = \frac{Frequency}{RotorSpeed}$$

3.1 Geometry

The blade was created by Solidworks, because it's easier and faster as the Ansys/Designmoddeler.

The model was designed as following figure 3-1.

- Create the inner radius surface
- Create the outer radius surface
- Create 2 planes with the right angle
- Create a 3D-Sketch
- Create a the blade surface

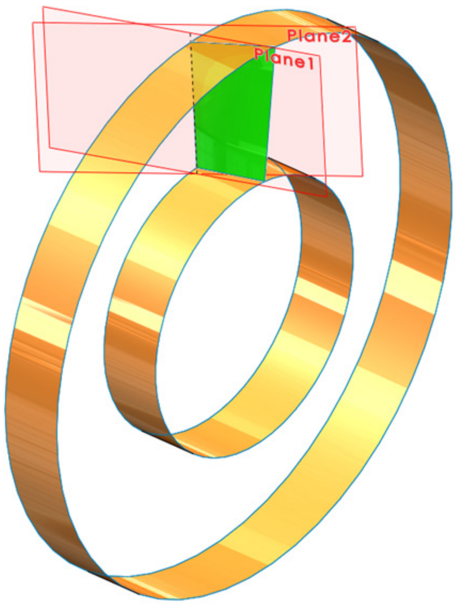


figure 3-1

3.2 Boundary Conditions

- Fixed support at the innermost radius
- Following rotor speeds are chosen: 0, 2400, 4800, 7200, 9600 and 12000RPM

3.3 Campbell-Diagram

In the figure 3-2 you can see the 4 different modes and the ratio of 1EO-4EO. Critical places are marked with a red circle. It's very important to figure out, that is no critical place near 9600RPM. In this case a tolerance of $\pm 800 RPM$, marked with a grey area, is chosen. In this area is no critical point.

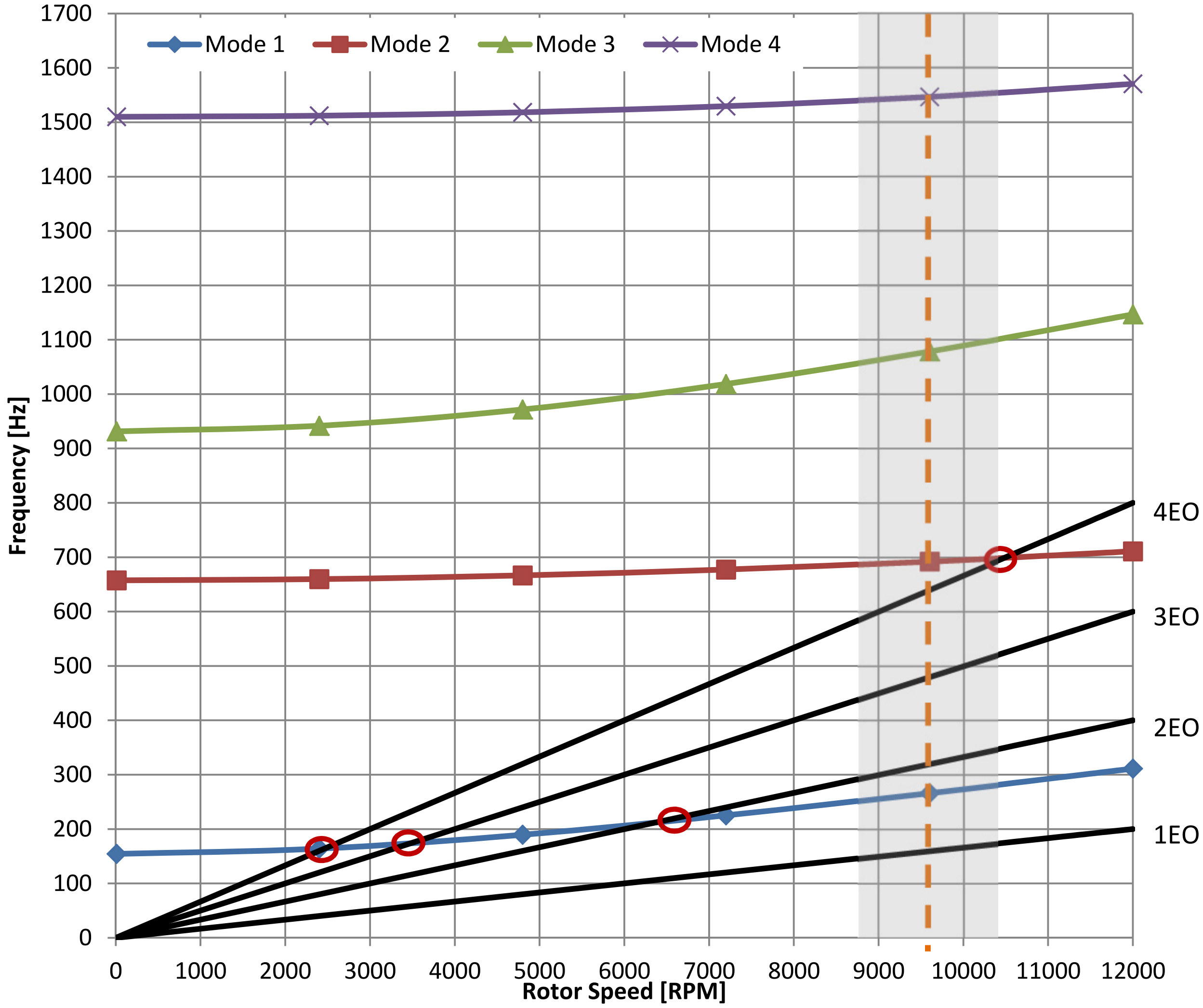
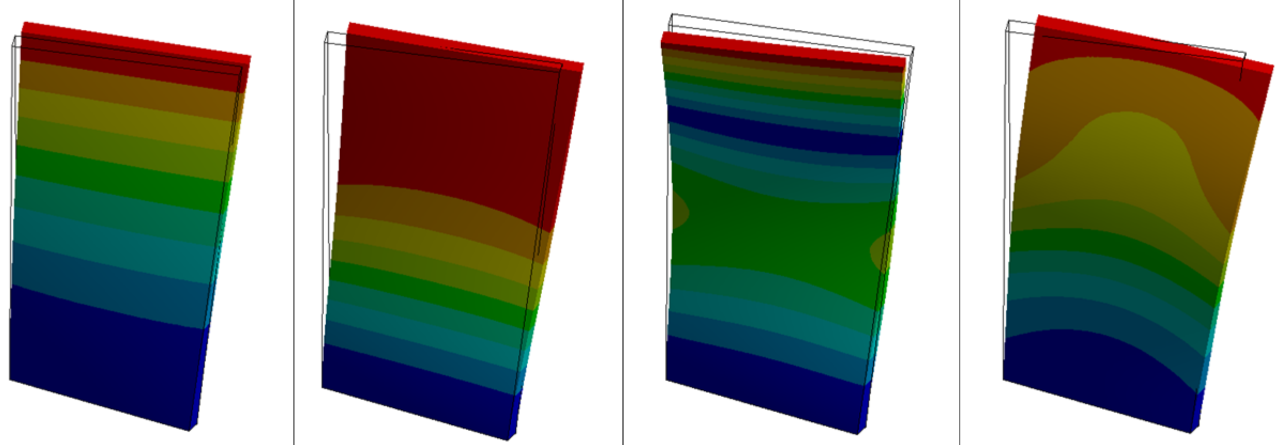


figure 3-2

3.4 Results

Mode 1	Mode 2	Mode 3	Mode 4



266.64 Hz	692.29 Hz	1078.5 Hz	1546.4 Hz

figure 3-3

4 DISCUSSION

The current geometry is very good and has no critical places near 9600RPM. It isn't necessary to change this geometry. After many iteration, I found a better geometry as the previous. figure 4-1 shows the new geometry without constant thickness with better results as in the previous geometry. As you can see in figure 4-2:

→Comparison:

- Only 3 critical places (marked red)
- The tolerances at 9600 RPM is increasing to $\pm 1400 RPM$ (marked grey)

However it's important to know, that's little variations in the thickness and the shape can lead to big changes concerning to the Eigen-value.

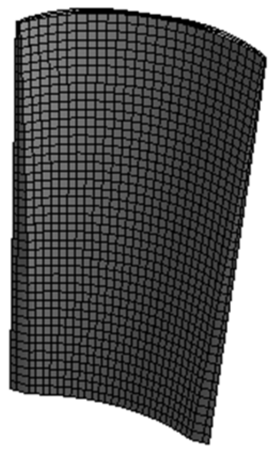


figure 4-1

By the way the Modal-Solution is a very good way to figure out under constrains in huge assemblies. If no supports (or partial) are present, rigid-body modes will occur at or near 0 Hz.

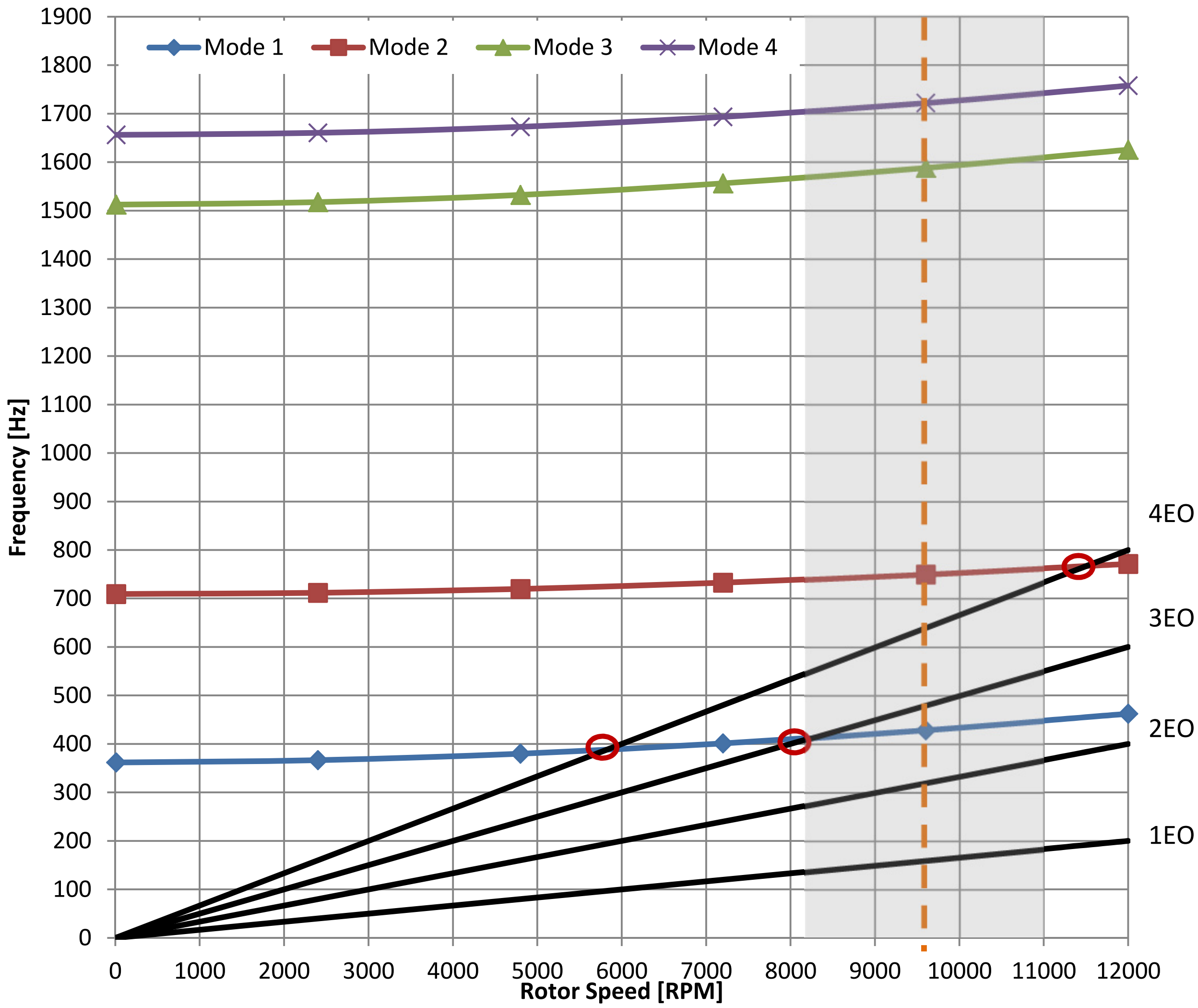


figure 4-2

4.1 Results of the final geometry

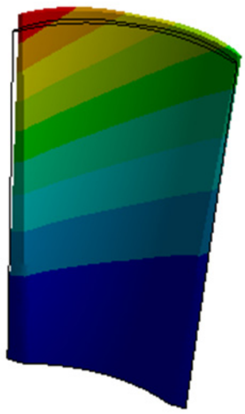
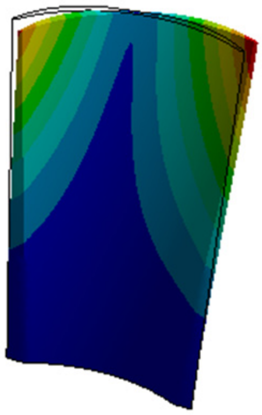
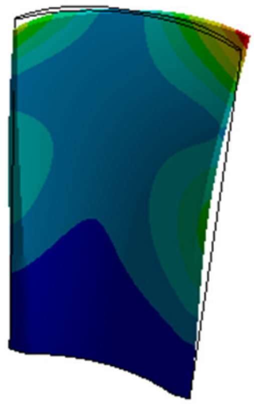
Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
			
428.8 Hz	749.33 Hz	1589.4 Hz	1719.7 Hz

figure 4-3

As you can see in figure 4-3, the next optimization was the upper edges of the blade.

5 APPENDIX

5.1 List of Figures

5.2 List of Tables